Advanced Placement Calculus Syllabus- BC

Prerequisites
• All students should have completed four years of secondary mathematics designed for accelerated students. These should consist of the accelerated or honors track in algebra, geometry, trigonometry, analytical geometry, and elementary functions.

• All students placed in BC Advanced Placement Calculus must have received a final average of an A- from Accelerated Pre-Calculus. An approved AP application is required. All AP applications are reviewed by a committee of accelerated math teachers. If a student is new to the school, they must take and receive the required grade on the departmental placement test in order to be placed in the course.

Technology requirement: It is essential that a student in this course own their own graphing calculator. Two sections of the AP exam require the use of one. Preferred: TI-83 Plus Texas Instruments graphing calculator (alternative: TI-84, 86 or 83).

Students should have an excellent understanding of the following topics:
1. Functions:
   linear, trigonometric, logarithmic, polynomial, piece-wise defined, exponential, parametric
2. Properties of Functions:
   inverses, inverse trig, degree, frequency, amplitude, period of trig functions without a calculator, asymptotes (partial understanding), composition of functions, absolute value (...and how to write them as a piece-wise function), slope of line, equation of a line in the form: \( y - y_1 = m(x - x_1) \), parallel, perpendicular, vertical, horizontal equations of lines
3. Algebra of Functions:
   synthetic division, simplifying with the conjugate, simplifying rational functions dividing functions w/long division
4. Graphs of Functions (without the use of any technology)
5. Language of Functions:
   domain and range, zeros, odd and even, periodic, symmetry(x-axis, y-axis, origin only), intercepts, etc.
6. Values of the trigonometric functions of common angles such as: \( 0, \pi/6, \pi/4, \pi/3, \) and \( \pi/2 \) without the use of a calculator.
Course Content

*Summer work is given to all students entering into BC Calculus.
*Prerequisite Quiz given within the first two weeks of school.
Prerequisite topics are listed above. A worksheet is reviewed beforehand.

A. Graphing Calculator Competencies  (2-3 days)
1. General Information
2. Graphing and Calculations
   a. Modes, Zoms, Windows, Table, Calc commands, Math commands
   b. Programming and Link
   c. Top Ten Common Problems and several practice exercises
   d. Graphing Calculator Lab: How does the TI graph?
      -Exploratory lab on how graphs can be interpreted, how pixels are connected and how to analyze functions prior to calculator utilization

B. Unit 1: Introduction to the Derivative  (5 weeks)
1. Average rate of change (slope of secant)
2. Instantaneous rate of change as a limit of the average rate of change
   a. Horizontal Tangents
   b. Displacement/Net Change
3. The Derivative
   a. Definition of the Derivative (Fermat’s Method)
   b. Derivative notation
   c. Where $\frac{df}{dx}$ does not exist
      -Graphing calculator exploration: Analyzing curves at differentiable and non-differentiable points
   d. Definition of differential calculus
   e. Absolute Value Derivatives
   f. Graphical relationships between $f(x)$ and its derivative
      -Graphing calculator exploration: $f$ and $f'$ graphs- discovering relationships
4. Derivative Applications
   a. Distance functions vs. Displacement
   b. Average Rate of Change/Average Velocity
   c. Velocity vs. Speed
   d. Instantaneous Velocity
   e. Acceleration
   f. Tangents and Normal Lines
   g. Graphing Distance, Velocity, and Speed
      -Writing assignment: Justifications given applications with position, velocity, speed
5. The Limit
   a. Definition of a Limit (Caunty)
   b. Finding Limits with a Graphing Calculator
      -Graphing calculator exploration: utilization of table and table set, appropriate viewing windows, value, correct modes
   c. Limit Combination Theorem
d. Right and Left Handed Limits  
   i. Piece-wise functions, Absolute Values and Greatest Integer Functions  

e. The Sandwich/Squeeze Theorem  
   -Graphing calculator exploration: visual interpretation of theorem with a compound inequality containing three curves, manipulation of viewing windows  

f. Trig Limits  
g. Infinity Limits  
h. Limit Substitution  

6. Continuity  
   a. Definition with Continuity Test  
   b. Theorems  
      i. Limit Combination Theorem for Continuous Functions  
      ii. Differentiable Functions and Continuity  
      iii. Max-Min Theorem for Continuous Functions  
      iv. Intermediate Value Theorem (examples include applying IVT when function is represented in tabular, graphical and analytical forms)  
   c. Continuous Extentions  
   d. Definition of Critical Points and Absolute extrema values  

7. Parametric, Polar, and Vector Functions  
   a. Analysis of Curves  

8. Major Assessments  
   a. Unit Test  
   b. AP Problem Set #1 (take-home test): collection of AP problems from past exams that pertain to the unit.  
   c. Written Assignment: Understanding of absolute extrema in application, tabular representation and graphical forms of functions.  
   d. Group presentations: Differentiation as an Application (must demonstrate differentiation in graphical, numerical, analytical forms, and communicate connections among these representations)  
   e. Group quiz on continuity  

C. Unit 2: Rules and Theorems of the Derivative  (5-6 weeks)  
1. Formulas for Taking the Derivative  
   a. Power Rule  
   b. Derivative of a Constant  
   c. Coefficient Rule  
   d. Sum Rule  
   e. Product Rule  
   f. Power Rule of a Differentiable Function  
   g. Quotient Rule  
   h. Second Derivative  

2. Implicit Differentiation  
   a. Power Rule for Fractional Exponents  
   b. Independent and dependent variables  
   c. Derivatives of Higher Order  

3. Derivatives of Trig Functions
4. Visualizing Solutions with Slope Fields
   a. Sketching of slope fields and predictions based on differential equations

5. Linear Approximations
   a. Linearization/Euler’s Method with differential equations
   b. Estimating Change with Differentials

6. The Natural Log
   a. Domain and Range
      - Graphing calculator exploration: transformations of the natural log, y = lnu
   b. Properties Review
   c. Derivative of the Natural log

7. Composition and Chain Rule
   a. Finding the derivative with three variables
   b. Composition and derivative given two graphs with three variables
   c. Parametric, Polar and Vector Functions
   d. 2nd Derivative with three variables
   e. Given function in terms of f and g, find derivative

8. Functions with “e”
   a. Graph, Domain and Range
   b. Changing Logs to Exponential Form, “solving for y” Review
   c. Derivative of e^x

9. Inverses and Their Derivatives
   a. Finding Inverses
   b. Derivative Rule for Inverses (analytical, tabular, and graphical functions are used in application)
   c. Inverse Trig Values and Derivatives

10. Major Assessments
    a. Section Quizzes: non-calculator and graphing calculator parts, open response graded with a college board rubric.
    b. Unit Test: Includes written portion- explanations/justifications of linear approximations in context of problem
    c. AP Problem Set #2(take-home test): collection of AP problems from past exams that pertain to the unit.
    d. Presentation: Smartboard utilization to describe differentiability of polar and parametric functions

D. Unit 3: Applications of the Derivative (4 weeks)
1. Curve Characteristics
   a. First Derivative Test
      i. Local and absolute extrema
      ii. Increasing and decreasing intervals
   b. Second Derivative Test
      i. Points of Inflection
      ii. Concavity intervals
   c. Second Derivative Test for Max/Mins
      - Given table of values, find local extrema by applying test
   d. Asymptotes(found with limits)
      i. L’Hopital’s Rule
e. Given table of values with $f'$ and $f''$, analysis of $f(x)$
f. Sketching of curves based on derivative test results and limits found.

2. Optimization
   - Graphing calculator exploration: Utilization of calculator commands to find optimization of rates.

3. Related Rates of Change
   Written assignment: Explanation of solutions using correct units of measure.

4. Theorems
   a. Rolle’s Theorem
   b. Mean Value Theorem (examples include applying IVT when function is represented in tabular, graphical and analytical forms)
   c. Max-Min Theorem for Continuous Functions (MVT Inequality)

5. Major Assessments
   a. Section Quizzes: non-calculator and graphing calculator parts, open response graded with a college board rubric.
   b. Unit Test
   c. AP Problem Set #3 (take-home test): collection of AP problems from past exams that pertain to the unit.
   d. Mid-Year Exam: Covers the first three units of the course and consists of four parts (same format as AP exam and graded with college board rubric)

E. Unit 4: The Anti-Derivative as a Summation (4 weeks)
1. General Solution vs. Particular Solution
2. Indefinite Integral Notation
3. Integration Formulas
   a. Power rule
   b. w/ Coefficients
   c. w/ addition and subtraction
   d. Direct substitutions, including transforming “u”, long division to alter integrand
   e. Integrations of Trig functions and Inverse Trig functions
   f. Integration by Parts
   g. Integration of Simple Partial Fractions (nonrepeating linear factors)
   - Cooperative learning assignment: the solutions of several different types of antiderivatives.
4. Separating Variables (solving all types of differential equations)
5. Finding “c” with initial conditions
6. Using acceleration to find velocity and to find position
7. The Definite Integral
   a. Approximating Definite Integrals
      i. Lower, Upper, Left, Right and Midpoint Riemann Sums
      ii. Trapezoidal Rule
      iii. Given a graph: Unequal sub-divisions, unknown $f(x)$ curves
   b. Algebraic Properties of Definite Integrals
   - Given a graph, interpretation of properties and evaluation of definite integrals
   c. Substitution with limits of integration
   d. Fundamental Theorems of Calculus
i. First FTOC
ii. Second FTOC (functions represented as a differentiable integration in application: analytical methods for simplification and evaluation)

-Written assignment: Explanation of the relationships between a definite and indefinite integration by applying FTOC.

e. Finding all types of areas
f. Improper Integrals
   i. L’Hospital’s Rule with convergence of improper integrals and series

8. Major Assessments
   a. Section Quizzes: non-calculator and graphing calculator parts, open response graded with a college board rubric.
   b. Unit Test
   c. AP Problem Set #4(take-home test): collection of AP problems from past exams that pertain to the unit.

F. Unit 5: Applications of the Anti-Derivative (3 weeks)
1. Distance/Velocity/ Acceleration Relationships
   a. Total Distance vs. Displacement
2. Area Between Curves
   a. “Top – Bottom”
   b. “Right – Left”
   -Graphing Calculator exploration: Finding areas and volumes using definite integral graphing calculator commands.
3. Volumes of Solids and Revolved Areas
   a. Volumes by slicing
   b. Volumes with given cross-sections
   c. Volumes with washers
   d. Volumes with shells
   -Written assignment: Given a rate application, explanation of how an accumulation is found with a definite integral
4. Average Value of a Function
5. Integrations of Exponential Functions and \( \ln u \)
   a. Integration of \( e^u \), \( \tan u \), \( \cot u \)
   b. Integration of \( a^u \) and \( \log_a u \)
6. Solving differential functions in applications
7. Relative Rates of Growth(comparisons of growths by using limits)
   -Graphing Calculator exploration: comparing the relative magnitudes of two functions
8. Use of the anti-derivative to solve a variety of problems in application(net accumulation of a rate, model a written description of a physical situation with an integral )
9. Major Assessments
   a. Section Quizzes: non-calculator and graphing calculator parts, open response graded with a college board rubric.
   b. Unit Test
   c. AP Problem Set #5(take-home test): collection of AP problems from past exams that pertain to the unit.
d. Smartboard presentation with calculus software: Visualizing the limiting process of the summation of volumes.

**G. Unit 6: Series and Polynomial Approximations (4-5 weeks)**

1. Concept of Series  
   a. definitions and examples (sequences, series)  
   b. decimal expansion examples  
   c. convergence/divergence of sequences  
   d. properties of infinite series

2. Geometric Series  
   a. sum with “a” and “r”  
   b. convergence of series  
   c. applications

3. Integral Test  
   a. series as an area of rectangles  
   b. use with an improper integral  
   c. $\rho$ series  
   d. harmonic series

4. Comparison Tests for Series  
   a. direct comparison test  
   b. limit comparison test

5. Alternating Series  
   a. test  
   b. error bound (remainder) and approximations

6. Ratio test

7. Taylor Series  
   a. Taylor and Maclaurin polynomial approximations of elementary functions  
      i. definitions of nth Taylor and Maclaurin polynomials  
      ii. examples and applications  
   b. Power Series  
      i. radius and interval of convergence  
   c. Definition of a Taylor and Maclaurin Series at $x = a$  
      \[
      \frac{1}{1-x}
      \]
   d. Maclaurin series for $\sin x$, $\cos x$, $e^x$, and $\frac{1}{1-x}$
   e. Finding and Using Taylor series  
      i. new series by substitution  
      ii. new series by differentiating and integrating  
      iii. applications  
   f. Lagrange error bound for Taylor Polynomials

8. Major Assessments  
   a. Section Quizzes: non-calculator and graphing calculator parts, open response graded with a college board rubric.  
   b. Unit Test

**H. Review for the AP Exam (2 weeks)**

1. Highlighting important information about the exam
2. Summaries of important concepts with AP problems to complete
3. Packet of Review Materials
   a. Vocabulary/concept fill-in
   b. Specific AP problems with answers
   c. Identifying types of problems
   d. How to prepare for the exam
   e. Review of any problem by request
4. Review Sessions(scheduled after school- usually the two days before the exam)

I. Technology Project (from AP Exam to End of Year)
   Two choices:
   Researching a new math topic OR Creating a lesson by incorporating math software/technology
   The project is graded on a four-point rubric based on the following criteria:
   technology, mathematics, visual graphics, depth of understanding, examples, level of difficulty,
   creativity, presentation of topics, collaborative work, and finished product.
   All projects are submitted in both electronic and hard copy form, and contain a written segment.
   All projects are presented (mostly done with the Smartboard technology)

* All students enrolled in the course are required to take the AP Calculus AB Exam in May. No
  student is allowed to drop the course after the first term.

⇒ AP Calculus is an intensive mathematics course. I have high expectations for my students
  that can be reached when a consistent, good effort is applied. I assess my students on the
  understanding of concepts rather than process and product. In return, students gain an
  appreciation of Calculus as a coherent body of knowledge and as a human accomplishment.